Guess My Rule

__________________ are lists of numbers where each term is based on the previous term or a combination of previous terms using a set pattern or rule.

Writing NOW-NEXT Rules for Sequences

Given the sequence of numbers: 3, 6, 9, 12, 15…

Step 1. The start is the number we begin with. In this case, the start number is ______.

Step 2. Write the rule in NOW-NEXT form. In the pattern above, each number increases by _____.

This means that the next number will be NOW + ______

Therefore, the rule should be written NEXT = NOW + ______. Start = _____

Try the following examples:

Determine the rule for the sequence and write it as a NOW-NEXT equation.

Example 1. 5, 10, 15, 20…

Example 2 2, 4, 8, 16, 32….

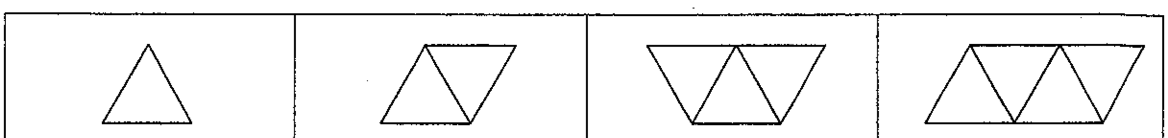
Example 3

Term	Value
1	-3
2	6
3	-12
4	24
5	-48

Example 4

Term	Value
1	52
2	46
3	40
4	34
5	28

Example 5



n = 1 P = 3	n = 2 P = 4	n = 3 P = 5	n = 4 P = 6

Common Core Math I
Recursive Patterns Practice

Name ________________________________

1) Consider the sequence of figures below made from triangles.

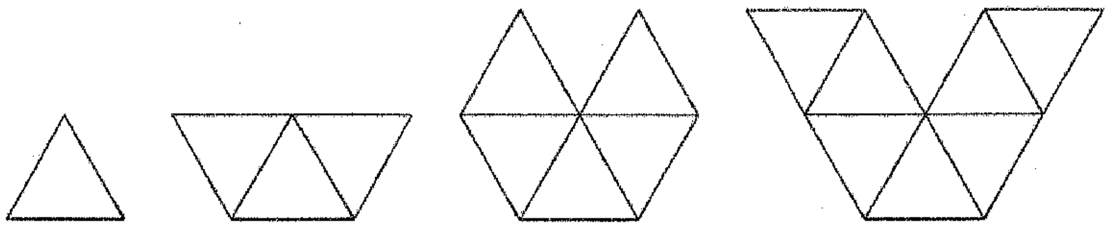


Figure 1 Figure 2 Figure 3 Figure 4

a) Complete the table below for the first five figures.

Figure Number	Perimeter
1	3
2	5
3	
4	
5	

b) Write a NOW-NEXT equation to find the perimeter of each figure.
c) Find the perimeter of the 10th figure.
d) Which number figure has a perimeter of 51?

2) List the first six values generated by the recursive routine below. Then write the routine as a NOW-NEXT equation.

-27.4 ENTER

Ans + 9.2 ENTER, ENTER, . . .

3) Write a NOW-NEXT equation for each sequence. Then use your equation to find the 8th term of each sequence.

a) 7.8, 3.6, -0.6, -4.8, . . .
b) -9.2, -6.5, -3.8, -1.1, . . .
c) 1, 3, 9, 27, . . .
d) 36, 12, 4, $\frac{4}{3}$, . . .

5

Common Core Math I Homework
Recursive Patterns

Name ______________________________

1) Solve each equation. Show all work!

a. $8(x-3)-9=-25$

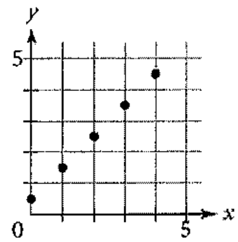
b. $16-5(x-4)=46$

c. $\dfrac{37-2(x+8)}{4}=4$

d. $\dfrac{-3(x-9)+4}{-4}=-10$

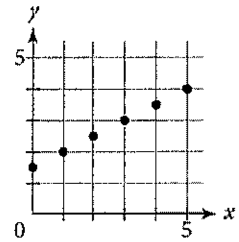
2) For each of the graphs below, fill in the table of values and write the NOW-NEXT equation for each relationship.

a.



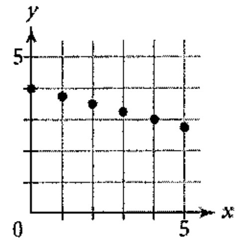
x	y

b.



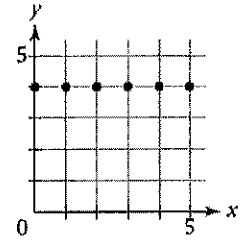
x	y

c.



x	y

d.



x	y

3) One hundred meter sticks are used to outline a rectangle. Write a recursive routine that generates a sequence of ordered pairs (l, w) that lists all possible rectangles.

4) Match the iterative routine in the first column with the equation in the second column.

a. 2 [ENTER]
Ans − 0.75 [ENTER], [ENTER], . . .

b. 0.75 [ENTER]
Ans + 2 [ENTER], [ENTER], . . .

c. −0.75 [ENTER]
Ans − 2 [ENTER], [ENTER], . . .

d. −2 [ENTER]
Ans + 0.75 [ENTER], [ENTER], . . .

1) START = 0.75
NEXT = NOW +2

2) START = -0.75
NEXT = NOW +2

3) START = 0.75
NEXT = NOW – 2

4) START = -0.75
NEXT = NOW – 2

5) START = 2
NEXT = NOW – .75

6) START = -2
NEXT = NOW – .75

7) START = 2
NEXT = NOW + .75

8) START = -2
NEXT = NOW + .75

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

In and Out Boxes

Fill in the Empty Boxes.

1)

In	Out
33	
35	
36	
40	

Rule: Add 9

2)

In	Out
33	
39	
43	
45	

Rule: Add 7

3)

In	Out
30	
31	
32	
44	

Rule: Add 10

4)

In	Out
30	
34	
35	
43	

Rule: Subtract 5

5)

In	32	35	43	44
Out				

Rule: Subtract 6

6)

In	30	31	38	45
Out				

Rule: Subtract 3

7)

In	33	37	39	43
Out				

Rule: Add 7

8)

In	34	37	41	45
Out				

Rule: Subtract 4

Write the rule and fill in the empty boxes.

9)

In	Out
30	26
38	
41	37
43	39

Rule:

10)

In	Out
33	43
37	
38	48
43	53

Rule:

11)

In	Out
33	34
37	
39	40
42	43

Rule:

12)

In	Out
32	24
35	
37	29
41	33

Rule:

13)

In	30	31	39	43
Out	28		37	41

Rule:

14)

In	30	35	37	45
Out	38	43	45	

Rule:

9

Testing for Functions

In this lesson you will
- represent relationships with tables, graphs, and equations
- use the **vertical line test** to determine whether a relationship is a function

You have written and used many rules that transform one number into another. For example, one simple rule is "Multiply each number by 2."You can represent this rule with a table, an equation, a graph, or a diagram.

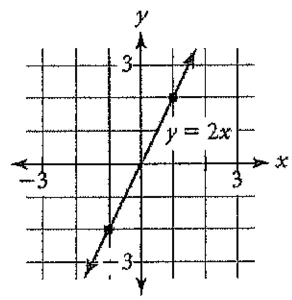
Table

Input x	Output y
3	6
−14	−28
9.3	18.6
x	$2x$

Equation

$y = 2x$

Graph



Diagram

Domain → Range

3 → 6

−14 → −28

9.3 → 18.6

In this lesson you will learn a method for determining whether a rule is a function either by applying the definition of function to graphs and tables.

Vocabulary:

Relation:

Function:

Domain:

Range:

Adapted from *Discovering Algebra: An Investigative Approach* by Murdock, Kamischke, and Kamischke

Investigation: Testing for Functions

In this investigation we will look at different representations of relationships - tables, algebraic statements (equations or inequalities), and graphs. In each case, we will decide whether the relationship represented is a function or not.

Part I: Tables

Table 1

x	y
1	-2
2	1
4	7
7	16
10	25

Table 2

x	y
1	-1
1	1
4	2
4	-2
9	3

Table 3

x	y
-2	4
-1	1
0	0
1	1
2	4

Table 4

x	y
2	-4
1	-1
0	0
1	1
2	4

Look at Table 1. Each input has only one output, so the relationship is a function.

In Table 2, the input values 1 and 4 each have two different possible outputs: the x-value 1 has corresponding y-values of -1 and 1, and the x-value 4 has corresponding y-values of 2 and -2. So Table 2 does not represent a function.

Table 3 represents a function and Table 4 does not. Explain why for each table.

Part II: Equations

Statement 1
$y = 2x + 1$

Statement 2
$y = \sqrt{x}$

Statement 3
$y = x^2$

Statement 4
$y < -2x + 4$

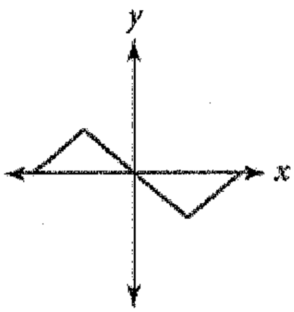
Consider Statement 1, $y = 2x + 1$. For any x-value that you input, you multiply by 2 and then add 1. There is only one possible output value that can result for any given input value. So Statement 1 represents a function.

For Statement 2, can you think of two different y-values that correspond to a single x-value? If $x = 4$, y can be 2 or -2, so Statement 2 does not represent a function.

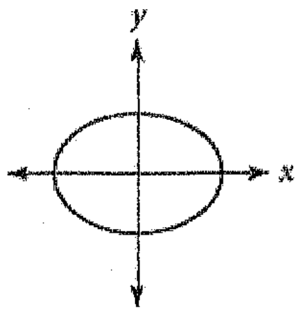
Statement 3 represents a function, and Statement 4 does not. Explain why for each statement.

Part III: Graphs

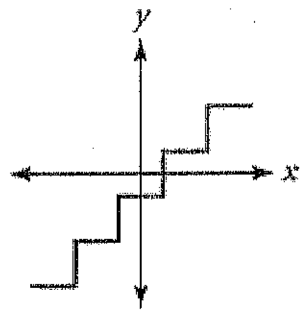
Graph 1



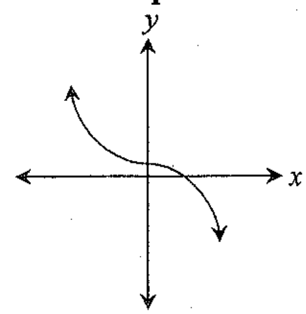
Graph 2



Graph 3

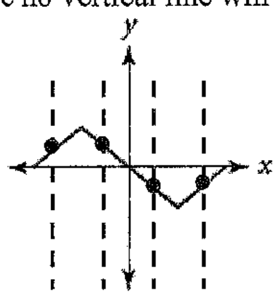


Graph 4

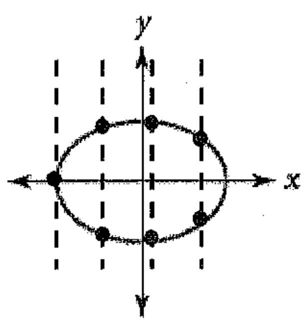


You can move a vertical line, such as the edge of a ruler, from left to right on a graph to determine whether the graph represents a function. If the vertical line ever intersects the graph in more than one point, you know that there is an x-value that has more than one corresponding y-value, so the graph is not a function.

Graph 1 represents a function because no vertical line will intersect the graph more than once.



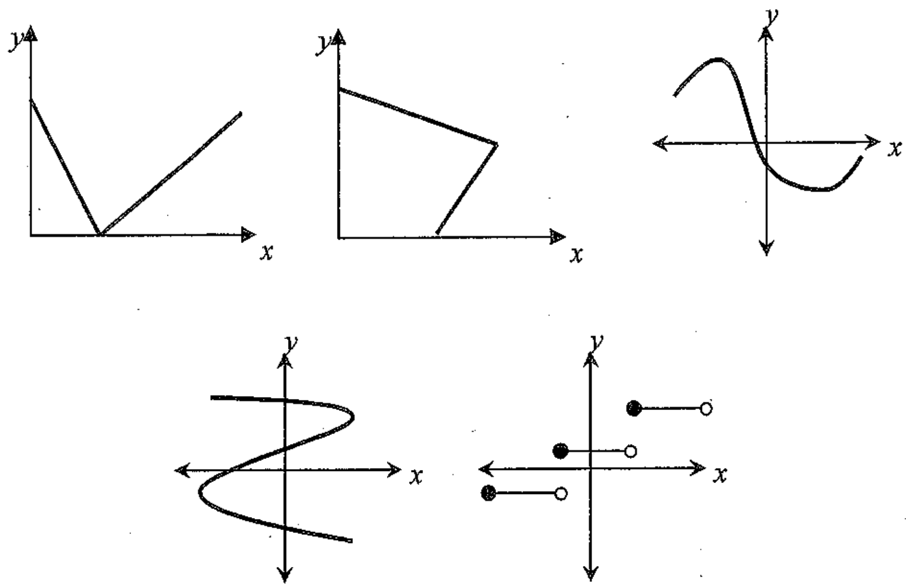
For Graph 2, however, 3 of the 4 vertical lines pictured intersect the graph twice. For each of these three x-values, there are two corresponding y-values, one positive and one negative, so the graph is not a function.



What about Graphs 3 and 4? For each graph, determine whether it is a function or not and explain why.

The **vertical line test** helps you determine whether a relationship is a function by looking at its graph. If all possible vertical lines cross the graph only once or not at all, then the graph is a function. If even one vertical line crosses the graph more than once, the graph is not a function.

EXAMPLE Use the vertical line test to determine which relationships are functions.



EXAMPLE Does each relationship of the form (input, output) represent a function? If the relationship does not represent a function, find an example of one input that has two or more outputs.

a) (city, zip code)
b) (person, birth date)
c) (last name, first name)
d) (state, capital)

Give an example of an (input, output) relationship that is a function.

Give an example of an (input, output) relationship that is not a function.

EXAMPLE Determine whether each table of x- and y-values represents a function. Explain your reasoning.

Input x	Output y
0	5
1	7
3	10
7	9
5	7
4	5
3	8

Input x	Output y
3	7
4	9
8	4
5	5
9	3
11	9
7	6

Input x	Output y
2	8
3	11
5	12
7	3
9	5
8	7
4	11

Name ______________________ Period ________ Date __________

Function Homework

1. Use the given equations to find the missing output values.

a. $y = 3 - x$

Input x	Output y
−4	
−3	
−2	
−1	
0	
1	
2	

b. $y = -1.5 + 3x$

Input x	Output y
−2	
−1.5	
−1	
−0.5	
0	
0.5	
1	

c. $y = 6.8 + 0.5x$

Input x	Output y
−6	
−2.4	
1	
2.8	
−14	
3.1	
−17.5	

2. Use the given equations to find the missing domain and range values.

a. $y = -3x + 5$

Domain x	Range y
−4	
−2	
	5
3	
	−7

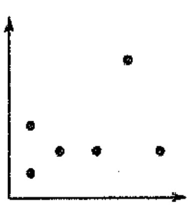
b. $2x - 3y = 6$

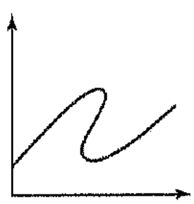
Domain x	Range y
	0
0	
	2
−6	
	5

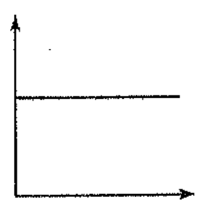
c. $x^2 - 2y = 11$

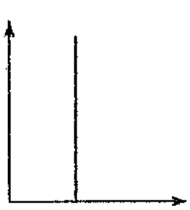
Domain x	Range y
−3	
0	
	7
1	
4	

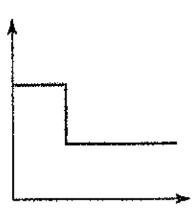
3. Find whether each graph represents a function.

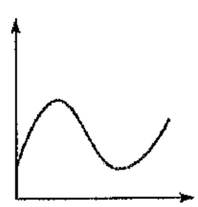
a.


b.


c.


d.


e.


f.


Function Homework cont.

4. On graph paper, draw a graph that is a function that satisfies the following conditions:

- the domain falls between -3 and 5
- the range falls between -4 and 4
- includes the points (-2, 3) and (3, -2)

5. On graph paper, draw a graph that is *not* a function that satisfies the following conditions:

- the domain falls between -3 and 5
- the range falls between -4 and 4
- includes the points (-2, 3) and (3, -2)

6. Complete the table of values for each question. Let x represent the input values and y represent the output values. Graph the points and determine whether or not the equation describes a function. Explain your reasoning.

a) $x - 3y = 5$

x	2		-4		0	
y		1		-2		0

b) $y = 2x^2 + 1$

x	-2	3	0	-3	-1	
y						9

Be careful when taking square roots – there are two possible values!

c) $x + y^2 = 2$

x	-7				-2	2
y		1	-2	-3		

d) $x + 2y = 4x$

x						
y						

7. Solve. Justify each step in your solution process with a mathematical property.

$$\frac{4(x-7)-8}{3} = 20$$

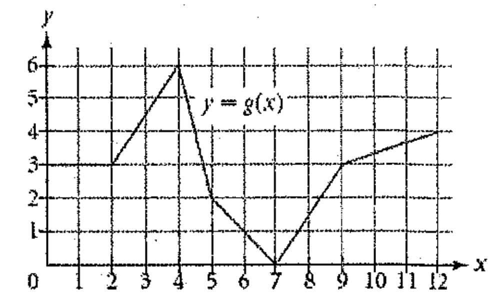
Function Notation

In this lesson you will

- learn to use **function notation**
- use a graph to evaluate a function for various input values
- use an equation to evaluate a function for various input values

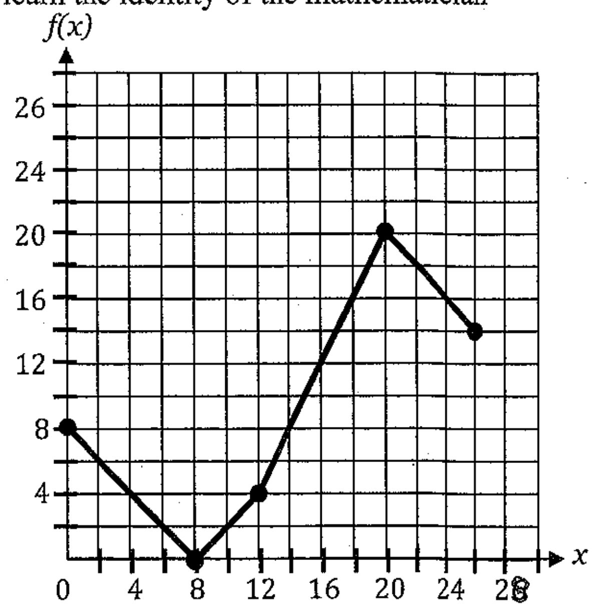
The equation $y = 1 - 2x$ represents a function. You can use the letter f to name this function and then use **function notation** to express it as $f(x) = 1 - 2x$. You read $f(x)$ as "f of x," which means "the output value of the function f for the input value x." So, for example, $f(2)$ is the value of $1 - 2x$ when x is 2, so $f(2) = -3$. (Note: In function notation, the parentheses do *not* mean multiplication.)

Not all functions are expressed as equations. Here is a graph of a function g. The equation is not given, but you can still use function notation to express the outputs for various inputs. For example, $g(0) = 3$, $g(4) = 6$, and $g(6) = 1$. Can you find x-values for which $g(x) = 3$?



Investigation: A Graphic Message

In this investigation, you will apply function notation to learn the identity of the mathematician who introduced functions. Look at the graph below.



What is the domain?

What is the range?

Use the graph to find each function value in the table. Then do the indicated operations. Show all work to the right.

Notation	Value
$f(3)$	
$f(18) + f(3)$	
$f(5) \times f(4)$	
$f(15) \div f(6)$	
$f(20) - f(6)$	

Function Notation pg. 2

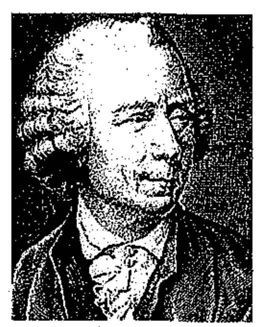
Think of the numbers 1 through 26 as the letters A through Z. Find the letters that match your answers in the table to learn the mathematician's last name.

Use the rules for order of operations to evaluate the expressions that involve function values. Do the operations inside parentheses first. Then find the function values before doing the remaining operations. Write your answers in the table. Show all work to the right of the table.

Notation	Value
$f(0) + f(3) - 1$	
$5 \cdot f(9)$	
x when $f(x) = 10$	
$f(9 + 8)$	
$\frac{f(17) + f(10)}{2}$	
$f(8 \cdot 3) - 5 \cdot f(11)$	
$f(4 \cdot 5 - 1)$	
$f(12)$	

Find the letters that match your answers in the table to learn the mathematician's first name.

This mathematician, who lived from 1707 to 1783, was a pioneering Swiss mathematician and physicist. He made important discoveries in calculus and graph theory. He also introduced much of the modern mathematical terminology and notation, including function notation. He is also renowned for his work in mechanics, fluid dynamics, optics, and astronomy. He spent most of his adult life in St. Petersburg, Russia and in Berlin (then a part of Prussia). He is considered to be the preeminent mathematician of the 18th Century, and one of the greatest of all time.*



* adapted from Wikipedia

Adapted from *Discovering Algebra: An Investigative Approach* by Murdock, Kamischke, and Kamischke

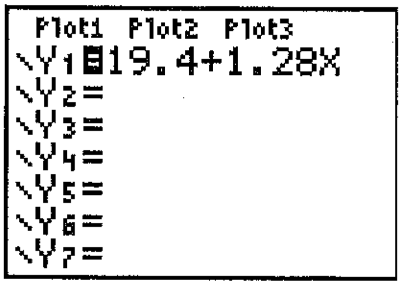
Function Notation p3

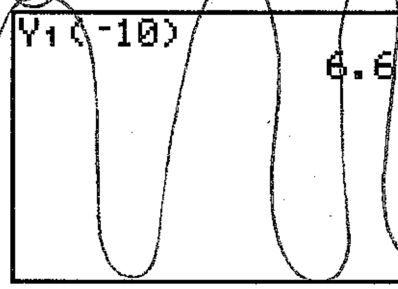
EXAMPLE You can use the function $f(x) = 19.4 + 1.28x$ to approximate the wind chill temperature $f(x)$ for a given actual temperature when the wind speed is 15 miles per hour. Both x and $f(x)$ are in degrees Fahrenheit. Find $f(x)$ for each given value of x.

a. $f(-10)$ **b.** $f(0)$ **c.** x when $f(x) = 19$ **d.** x when $f(x) = -13$

To evaluate functions on your calculator, enter the function into Y=. In this case, enter 19.4+1.28X into Y_1. Your calculator uses the notation $Y_1(X)$ instead of $f(x)$. To enter a statement using function notation, go to the home screen by hitting [2nd] [MODE] to Quit. To calculate the value for part *a* above, $f(-10)$, we must enter $Y_1(-10)$. To find Y_1, hit ❺, arrow right to Y-VARS, select option 1: Function, and then select option 1: Y_1. Back at the home screen, complete the calculation by typing (-10) and hitting [ENTER].

Y=





2ND GRAPH
Scroll to X and Y1 values.
2ND WINDOW

Note that the calculator cannot reverse the process for you, as in parts *c* and *d*. The calculator can only evaluate a function for a given input.

Unlike the calculator, when you write an equation for a function, you can use any letters you want to represent the variables and the function. For example, you might use

for the wind chill function discussed above.

EXAMPLE Use the graph of $y = f(x)$ at the right to answer each question.

a) What is the value of $f(4)$?

b) What is the value of $f(6)$?

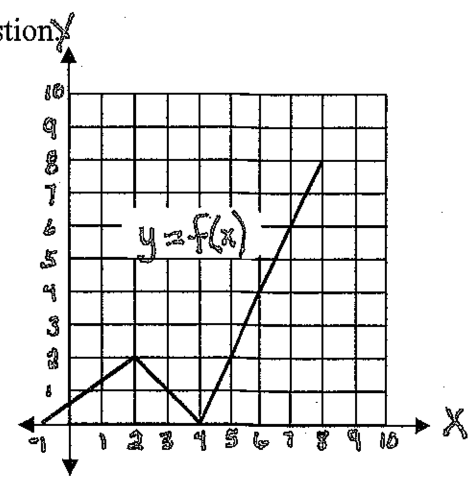
c) For what value(s) does $f(x) = 2$?

d) For what value(s) does $f(x) = 1$?

e) How many x-values make the statement $f(x) = 0.5$ true?

f) For what x-values is $f(x)$ greater than 2?

g) What are the domain and range shown on the graph?



Adapted from *Discovering Algebra: An Investigative Approach* by Murdock, Kamischke, and Kamischke

Function Notation p. 4

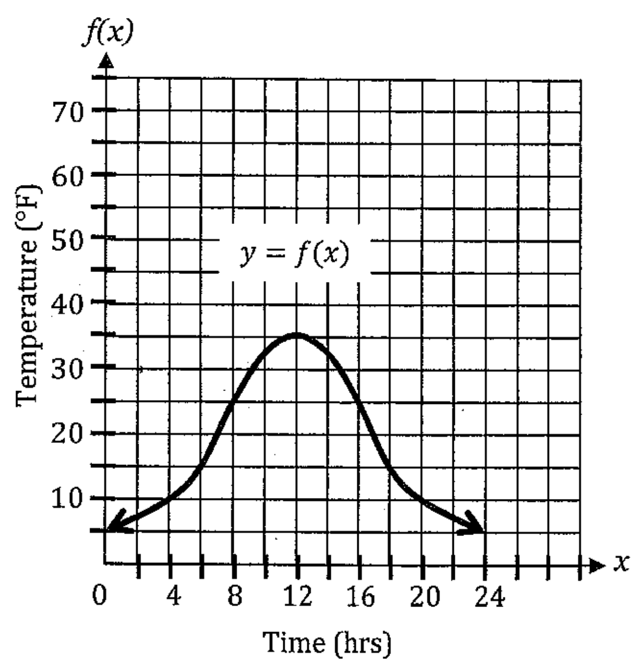
EXAMPLE The graph of $y = f(x)$ below shows the temperature y outside at different times x over a 24-hour period. Use the graph to answer each question.

a) What are the dependent and independent variables?

b) What are the domain and range shown on the graph?

c) Use function notation to represent the temperature at 10 hours.

d) Use function notation to represent the time at which the temperature is 10°F.



Algebra I
Function Notation Worksheet

Name: ______________________________
Hour: _________ Date: _______________

1. Evaluate the following expressions given the functions below:

$g(x) = -3x + 1$ $f(x) = x^2 + 7$ $h(x) = \frac{12}{x}$ $j(x) = 2x + 9$

a. $g(10) =$

b. $f(3) =$

c. $h(-2) =$

d. $j(7) =$

e. $h(a)$

f. Find x if $g(x) = 16$

g. Find x if $h(x) = -2$

h. Find x if $f(x) = 23$

i. CHALLENGE! (in other words, optional) $g(b+c)$

j. CHALLENGE! (also optional) $f(h(x))$

2. Translate the following statements into coordinate points:

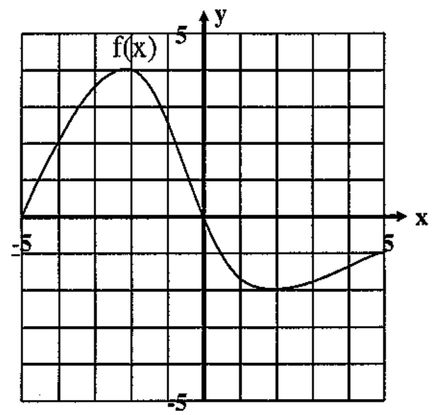
a. $f(-1) = 1$

b. $h(2) = 7$

c. $g(1) = -1$

d. k(3) = 9

3. Given this graph of the function f(x):



Find:

a. f(–4) =

b. f(0) =

c. f(3)

d. f(-5)

e. x when f(x) = 2

f. x when f(x) = 0

Name ______________________ Period __________ Date ______________

Function Notation Homework

1. Find each function value for $f(x) = 4x - 7$ and $g(x) = -3x + 5$ without using your calculator. Then enter the equation for $f(x)$ into Y1 and the equation for $g(x)$ into Y2. Use function notation on your calculator to check your answers.

a. $f(2)$ b. $f(0)$ c. $f(-3)$ d. $g(1)$

e. $g(6)$ f. $g(-7)$ g. $f(0.5)$ h. $g(0.5)$

i. $f(3.25)$ j. $g\left(\frac{2}{3}\right)$ k. $f\left(-\frac{11}{8}\right)$ l. $g(-2.3)$

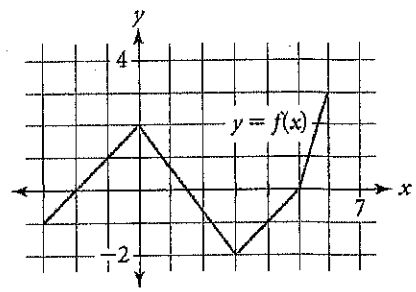
2. Find the y-coordinate corresponding to each x-coordinate if the functions are $f(x) = 2x^2 + x - 3$ and $g(x) = 40(0.8)^x$. Check your answers with your calculator.

a. $f(1)$ b. $f(-3)$ c. $f(0)$ d. $f(4)$

e. $f(-0.5)$ f. $g(1)$ g. $g(0)$ h. $g(-2)$

i. $g(3)$ j. $g(-1)$

3. Use the graph of $y = f(x)$ to answer each question.



a. What is the value of $f(0)$?

b. What is the value of $f(3)$?

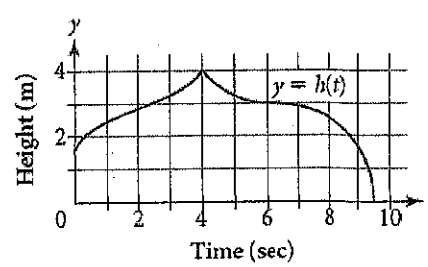
c. For what x-value or x-values does $f(x)$ equal 3?

d. For what x-value or x-values does $f(x)$ equal 0?

e. For what x-values is $f(x)$ less than 0?

f. What are the domain and range shown on the graph?

4. The graph of the function $y = h(t)$ shows the height of a paper airplane on its maiden voyage.



a. What are the dependent and independent variables?

b. What are the domain and range shown on the graph?

c. Use function notation to represent the plane's height after 6 seconds.

d. Use function notation to represent the time at which the plane was 4 meters high.

5. The function $f(x)$ gives the lake level over the past year, with x measured in days and $f(x)$, or y, measured in inches above last year's mean height.

a. What is the real-world meaning of $f(60)$?

b. What is the real-world meaning of $f(x) = -3$?

c. What is the interpretation of $f(20) = f(150)$?

13

6. The graph shows part of the function $f(x) = 500(0.80)^x$.

   a. What is the dependent variable and what are its units?

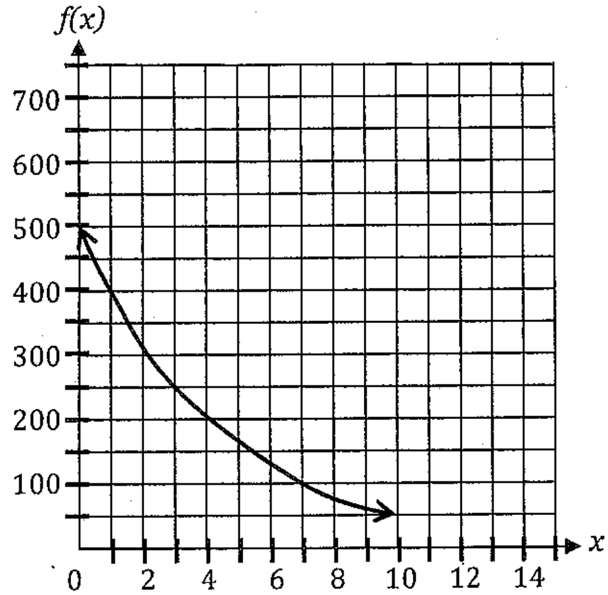
   b. What is the independent variable and what are its units?

   c. What part of the domain is pictured? What is the domain of the function?

   d. What part of the range is pictured? What is the range of the function?

   e. What is $f(0)$ for this graph?

   f. Find the value of x when $f(x) = 200$.



Name__________________

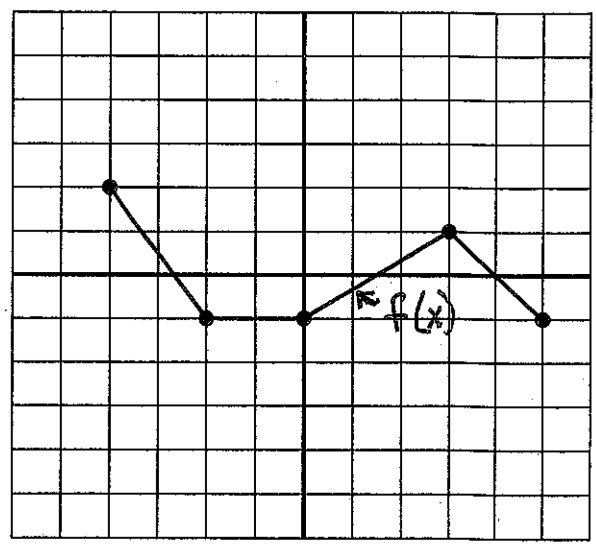
Transformations of Functions: Horizontal and Vertical Translations

In this lesson you will
- identify the effect on the graph of replacing $f(x)$ by $f(x+h)$ and $f(x)+k$.
- find the value of h or k (the amount and direction of translation) given the graph of a parent function and its transformed image.

A **translation** is an operation that shifts a graph horizontally, vertically, or both. The result is a graph that is the same shape and size, located in a different position. The variables h and k are commonly used to represent the general form of a translation. Vertical translations are represented by the value for k, while h is the variable used to represent horizontal translations. The original function $f(x)$ is called the **parent function.** By transforming a parent function, you can create infinitely many functions in the same **family of functions.**

Investigation: Vertical Translations of Functions $f(x)+k$

Consider the graph of $f(x)$ below. To do a vertical translation of a function presented as a graph, simply translate, or move, every point on the parent graph up or down k units. Let's do the following translation: $f(x)+3$



What is the value of k?

Should we move the graph up or down?

How many units should we move each point?

Translate and draw $f(x)+3$ on the same graph as $f(x)$.

Selected points on the parent graph $f(x)$ are recorded in the table below. Record the y-values of the corresponding points of $f(x)+3$ in the table.

$f(x)$

x	y
-4	2
-2	-1
0	-1
3	1
5	-1

$f(x)+3$

x	y
-4	
-2	
0	
3	
5	

What do you notice about the corresponding y-values?

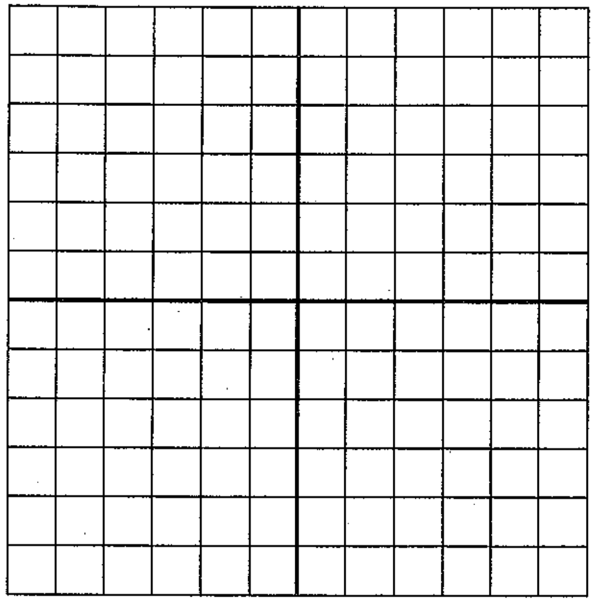
Use what you have just discovered to fill in the y-values for $g(x) - 4$. Then graph both $g(x)$ and $g(x) - 4$ on the same set of axes using different colors. Label each function.

$g(x)$

x	y
-5	2
-3	3
0	2
1	1
2	-2

$g(x) - 4$

x	y
-5	
-3	
0	
1	
2	



What is the value of k?

How does the second graph compare to the parent function?

Based on what you have seen, fill in the following:

When the value of k is ____________________, the parent graph is translated k units up.

When the value of k is ____________________, the parent graph is translated k units down.

Investigation: Horizontal Translations of Functions $f(x + h)$

Consider the parent function $f(x) = 2x + 3$. Let's do the following translation: $f(x + 2)$

To do this algebraically, we would replace "x" in the parent function $f(x)$ with "$x + 2$":

$$f(x) = 2x + 3 \rightarrow f(x + 2) = 2(x + 2) + 3$$

So our new function is:

$$f(x + 2) = 2(x + 2) + 3$$

This function can be simplified by using the distributive property and combining like terms:

$$f(x + 2) = 2x + 4 + 3$$
$$f(x + 2) = 2x + 7$$

Let's see what this looks like in table and graph forms.

First, find the corresponding y-values for the selected x-values in the first table below.

Then find the corresponding **x-values** for the selected y-values in the second table by using the equation and working backwards.

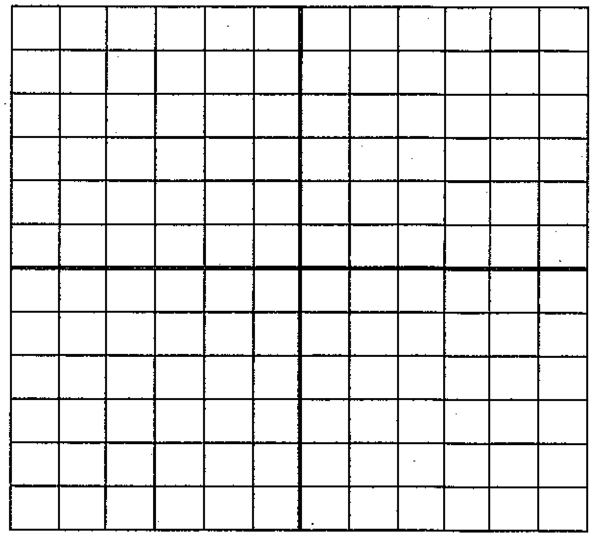
Then graph each function in a different color on the same set of axes. Be sure to label each function.

$f(x) = 2x + 3$

x	y
-3	
-2	
0	
1	

$f(x+2) = 2x + 7$

x	y
	-3
	-1
	3
	5



Was the parent function translated to the left or to the right?

What is the value of h?

How are the x-values in the two tables related?

How are the y-values in the two tables related?

Now consider the parent function $g(x) = |x|$. This is the **absolute value function.** For each value of x, the corresponding y-value is the distance of the x-value from zero on the number line. For example, $g(-4) = |-4| = 4$, since -4 is 4 units from zero on the number line.

Fill in the values for $g(x)$ in the table on the next page and graph the points. Then finish graphing $g(x)$ by connecting the points to form a "V" shape.

Next, let's graph a translation of the absolute value function: $g(x-3)$

Algebraically speaking, $g(x-3) = |x-3|$.

For the second table, find the corresponding y-values for the selected x-values.

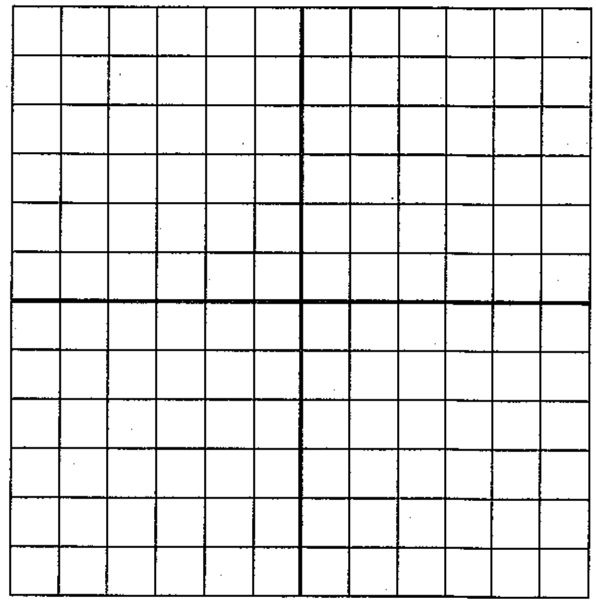
Then graph the translated function in a different color on the same set of axes. Be sure to label each function.

$g(x) = |x|$

x	y
-3	3
-2	
-1	1
0	
1	
2	2
3	
4	
5	5

$g(x-3) = |x-3|$

x	y
0	
1	
2	
3	
4	
5	
6	
7	
8	



Was the parent function translated to the left or to the right?

What is the value of h?

How are the x-values in the two tables related?

How are the y-values in the two tables related?

Based on what you have seen, fill in the following:

When the value of h is ____________________, the parent graph is translated h units left.

When the value of h is ____________________, the parent graph is translated h units right.

Vertical and Horizontal Translations of Functions

Name ____________

1) Describe each translation:

	Parent Function	Translated Function
a)	$y = \|x\|$	$y = \|x + 4\|$
b)	$y = \frac{1}{2}x - 6$	$y = \frac{1}{2}(x - 4) - 6$
c)	$y = \|x\|$	$y = 1 + \|x - 3\|$
d)	$y = -\frac{3}{4}x + 1$	$y = -\frac{3}{4}x + 8$

2) Write the function $t(x)$ for each of these transformations:

a) Translate the graph of $f(x) = |x|$ left 5 units.

b) Translate the graph of $f(x) = 2x - 7$ right 2 units.

c) Translate the graph of $f(x) = |x|$ down 4 units.

d) Translate the graph of $f(x) = -3x + 2$ left 2 units and up 3 units.

3) Is the following function a translation 5 units right or 5 units down? Explain

Parent function: $f(x) = x$ Translated function: $t(x) = x - 5$

4) Below are tables of points for two functions. Describe the transformation.

Parent function

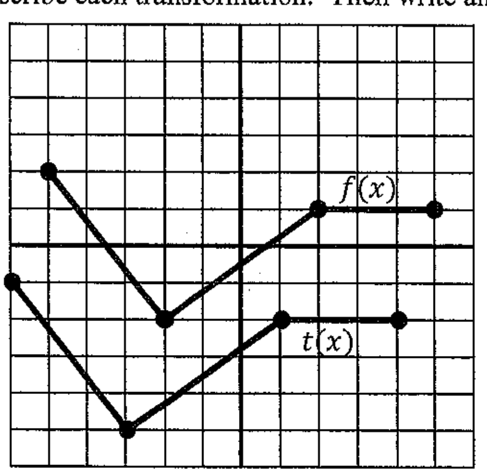
x	y
−1	3
3	5
2	4

Translated function

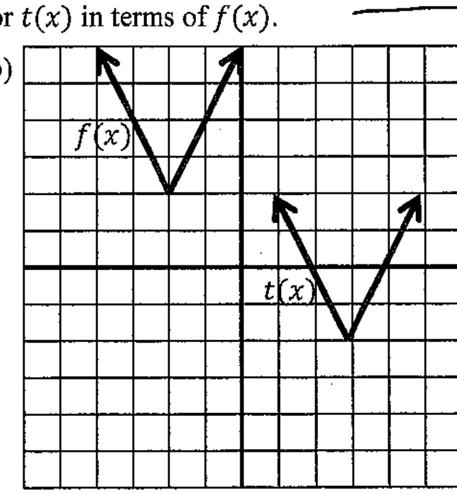
x	y
7	−1
11	1
10	0

5) Describe each transformation. Then write an equation for $t(x)$ in terms of $f(x)$.

a)



b)



1a. ____________

b ____________

c. ____________

d. ____________

2a ____________

b ____________

c. ____________

d. ____________

3. ____________

____

____

4. ____________

____

5. a. t(x)= ____________

____

____

____

b. f(x) = ____________

____

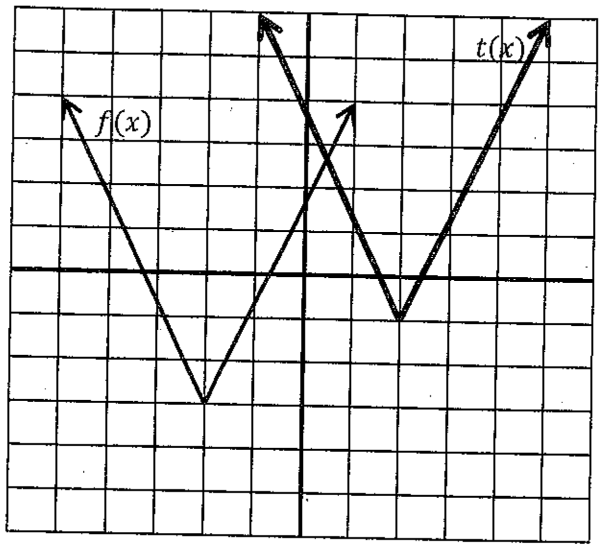
____

____

Combinations of Vertical and Horizontal Translations of Functions

EXAMPLE Examine the equation for each parent graph, table, or equation (labeled $f(x)$) and its corresponding transformed graph, table, or equation (labeled $t(x)$). Describe the translation.

a)



b)

$f(x)$

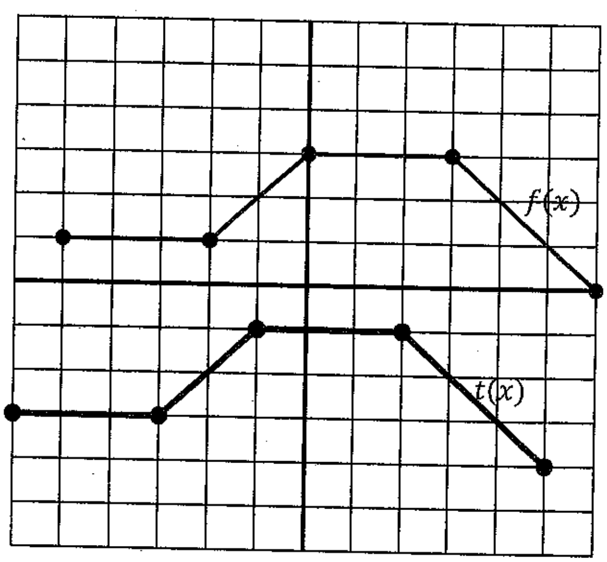
x	y
-2	4
-1	1
0	0
1	1
2	4
3	9

$t(x)$

x	y
-2	10
-1	7
0	6
1	7
2	10
3	15

c) $t(x) = f(x) - 7$

d)



e) $t(x) = f(x + 5)$

f) $t(x) = f(x - 8) + 3$

Rate of Change and Average Rate of Change Name ___________

In this lesson you will

- estimate the rate of change from a graph.
- calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Every day we deal with quantities expressed as ratios: miles per gallon of gas, cost per kilowatt of power, miles per hour that a car is travelling. When working with functions that relate two quantities such as miles and gallons or cost and kilowatts or miles and hours, we refer to these ratios as **rate of change**. Rate of change tells us how much one quantity is changing with respect to another quantity. For example, a speed of 60 mph tells us that a vehicle travels 60 miles for each hour it is driven.

Some rates of change are constant, and others are not. For example, if a car travels from one city to another, it does not normally travel at a constant rate. The car will speed up or slow down depending on traffic, or may stop for a period of time so the driver and passengers can grab a bite to eat. When the rate is not constant, we often look at the **average rate of change**. The average rate of change tells us how much one quantity changes with respect to another quantity over a specified interval. So if the car travels 150 miles in 3 hours (**rate of change**), we can say that the average rate of change (or speed) that the car travelled was 50 miles per hour (per 1 of a unit).

"Eureka!"

This exclamation is most famously attributed to the ancient Greek scholar Archimedes. He reportedly proclaimed "Eureka!" when he stepped into a bath and noticed that the water level rose—he suddenly understood that the volume of water displaced must be equal to the volume of the part of his body he had submerged. He then realized that the volume of irregular objects could be measured with precision, a previously intractable problem. He is said to have been so eager to share his discovery that he leapt out of his bathtub and ran through the streets of Syracuse naked.

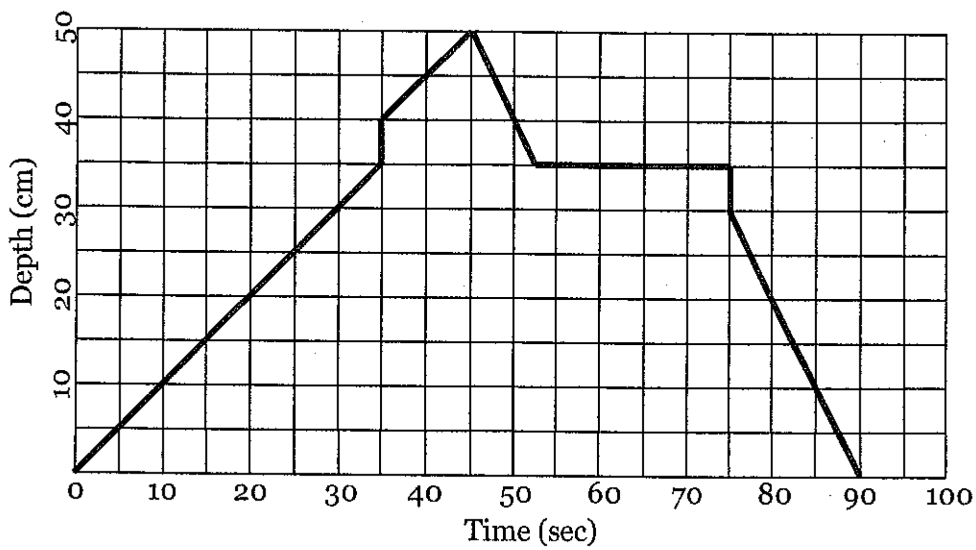
Archimedes' insight led to the solution of a problem posed by Hiero of Syracuse on how to assess the purity of an irregular golden crown; he had given his goldsmith the pure gold to be used, and correctly suspected he had been cheated by the goldsmith removing gold and adding the same weight of silver. Equipment for weighing objects already existed, and now that Archimedes could also measure volume, their ratio would give the object's density, an important indicator of purity. (Gold and silver do not have the same density, so if the density measured was not the same as the density of gold, they would know that the goldsmith was cheating.)[1]

[1] Adapted from http://en.wikipedia.org/wiki/Eureka_(word)

Investigation: Rae of Change and Graphs

See what happens with different tap levels, when the tap is on or off, when the plug is in or out, and when Archimedes in the bath and not in the bath.

Consider the following graph for Archimedes Bathtub.



Answer the following questions using the graph.

1) What happens when the water reaches a depth of 50 cm? How is this reflected on the graph?

2) What would happen to the depth when Archimedes gets in the bathtub? How is this reflected on the graph?

3) What happens to the depth when the tap is off and the plug is in? How is this reflected on the graph?

4) What conditions exist when the graph is increasing?

5) What conditions exist when the graph is decreasing?

6) What is the rate of change of the depth of the water from 0 to 30 seconds? Is this a constant rate? How can you tell?

7) What is the rate of change of the depth of the water from 35 to 45 seconds? Is this a constant rate? How can you tell?

8) What is the rate of change of the depth of the water from 45 to 52 seconds? Is this a constant rate? How can you tell?

9) What is the rate of change of the depth of the water from 55 to 70 seconds? Is this a constant rate? How can you tell?

10) What is the rate of change of the depth of the water from 70 to 75 seconds? Is this a constant rate? How can you tell?

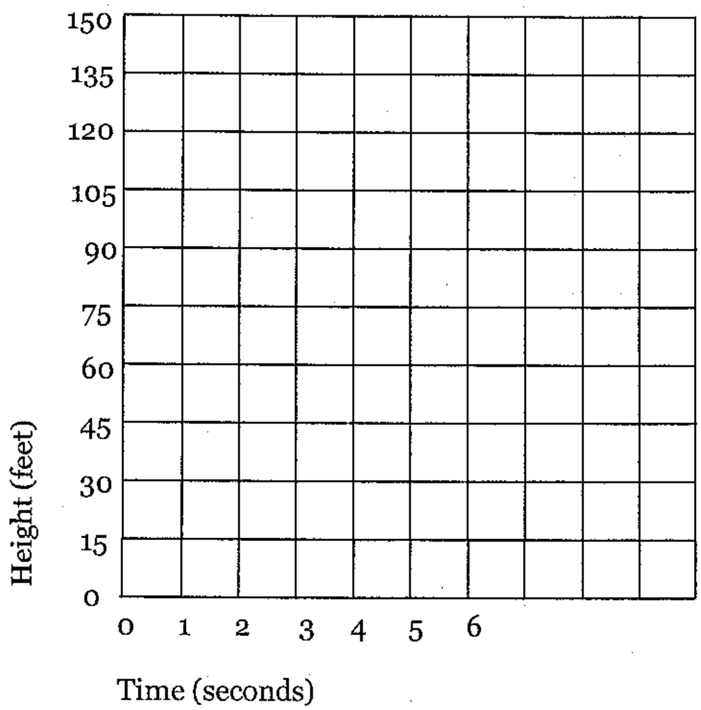
11) What is the rate of change of the depth of the water from 80 to 90 seconds? Is this a constant rate? How can you tell?

12) Is this relationship a function? Why or why not?

Investigation: Average Rate of Change

The height, h, in feet, of a ball thrown into the air is modeled by the function $h(t) = -16t^2 + 96t$, where the time, t, is measured in seconds. Fill in the table below and sketch a graph of the function on the grid provided by connecting the points with a smooth curve.

t	$h(t)$
0	
1	
2	
3	
4	
5	
6	



How far did the ball travel from 0 to 2 seconds?

What is the average rate of change (distance traveled per second) for the first two seconds?

How far did the ball travel from 2 to 4 seconds?

What is the average rate of change (distance traveled per second) for these two seconds?

Is the ball travelling faster during the first 2 seconds or the next 2 seconds? How can you tell? How is this revealed in the graph?

Name:_______________________________

Rate of Change Homework
Common Core Math I Unit 3

Date:____________________Period:____

1) Tamara made several deposits to her savings account over the summer as she was working. Her balance increased from $1140 on her May bank statement to $1450 on her September bank statement. Find the average rate of change per month. Round your answer to the nearest dollar.

2) Jocelyn is a receptionist at a doctor's office. She tracked the average waiting time at the office each month for five months.

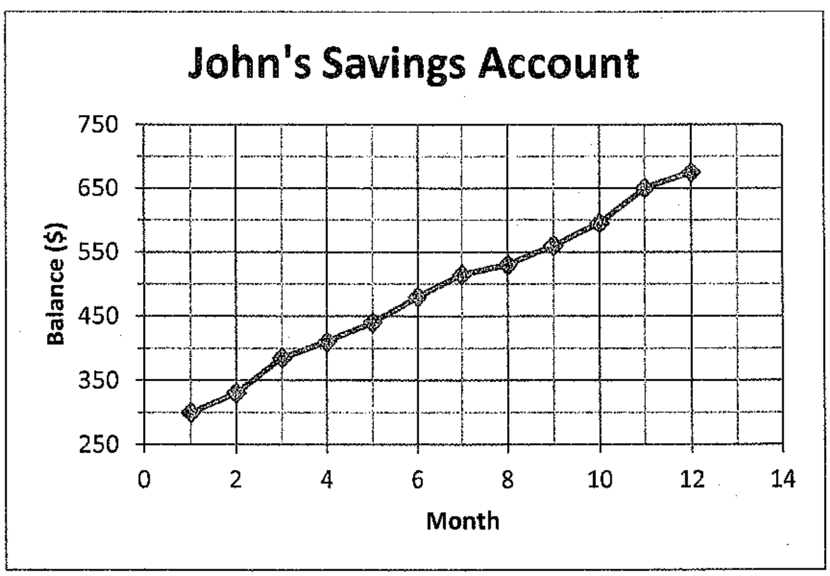
Month	Waiting Time (minutes)
January	25
February	22
March	6
April	16
May	10

a) What is the rate of change between April and May?

b) What is the average rate of change from January to May?

c) What might explain the lower average waiting time for March as compared to the other months?

3) John would like to find out how much money he saved per month for the year. Below is a graph showing the balance each month. Find the approximate average rate of change for the year.



4) The population of Raleigh rose from 290,000 in 2000 to 416,000 in 2011. Find the average rate of change in population (people per year) for this time period.

5) In the Mojave Desert in California, temperatures can drop quickly from daytime to nighttime. Suppose the temperature drops from 100°F at 2:00 pm to 68°F at 5 am. Find the average rate of change in temperature for this time period.

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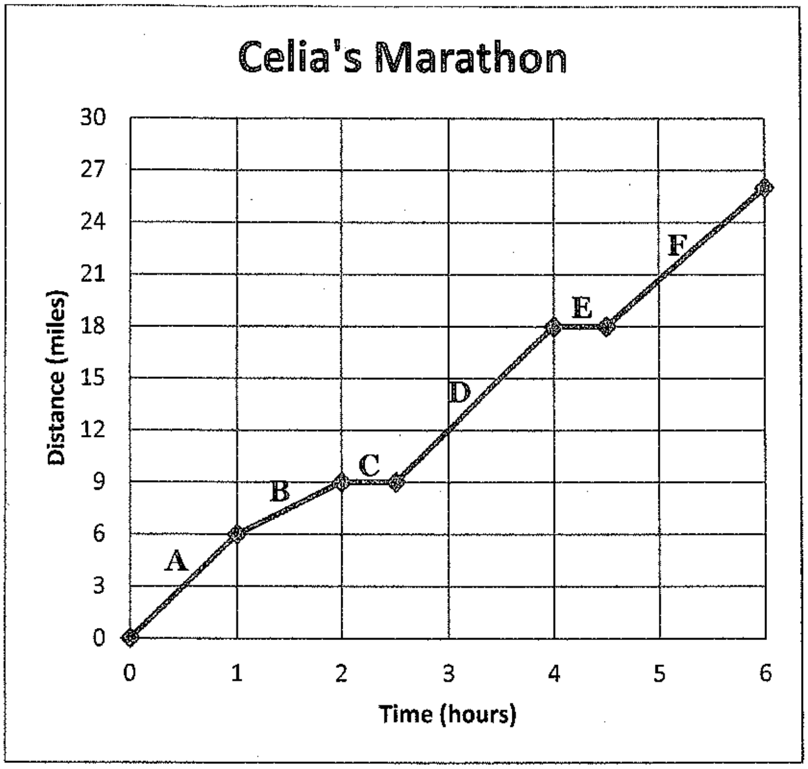
6) The following graph represents Celia's marathon race.



a) Find the rate of change in miles per hour for each interval.

Interval	Miles	Hours	MPH
A			
B			
C			
D			
E			
F			

b) During which interval(s) was Celia running the fastest? How do you know?

c) During which interval(s) was Celia resting? How do you know?

d) If Celia maintained the same rate of change as interval A for the entire marathon, how long would it have taken her to finish the race? (A marathon is 26 miles long.)

7) Lucille bought a house in 1998 for $144,000. In 2012, the house is worth $245,000. Find the average rate of change in dollars per year of the value of the house. Round your answer to the nearest dollar.

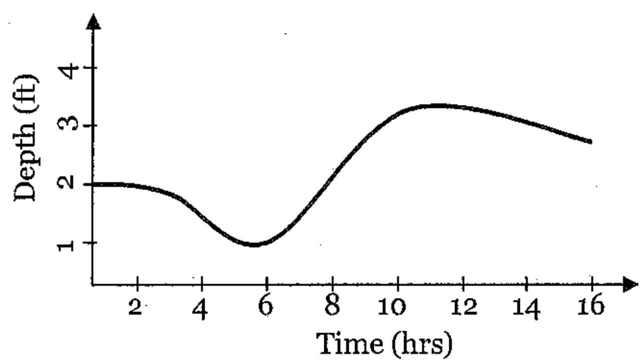
Graphs of Real-World Situations

In this lesson you will
- describe graphs using the words **increasing, decreasing, linear**, and **nonlinear**
- match graphs with descriptions of real-world situations
- learn about **continuous** and **discrete** functions
- use intervals of the domain to help you describe a function's behavior

Like pictures, graphs communicate a lot of information. So you need to be able to draw and make sense of graphs. In Unit 1, you learned to interpret dotplots, histograms, and boxplots based on one quantity. In this lesson you'll look at graphs that show how two real-world quantities are related, and you'll practice interpreting and describing graphs.

Investigation: Interpreting Graphs

This graph shows the relationship between time and the depth of water in a leaky swimming pool.



What is the initial depth of the water?

For what time interval(s) is the water level decreasing? What accounts for the decrease(s)?

For what time interval(s) is the water level increasing? What accounts for the increase(s)?

Is the pool ever empty? How can you tell?

In this example, the depth of the water is a function of time. That is, the depth depends on how much time has passed. So, in this case, depth is called the **dependent variable**. Time is the **independent variable**. When you draw a graph, put the independent variable on the x-axis and put the dependent variable on the y-axis.

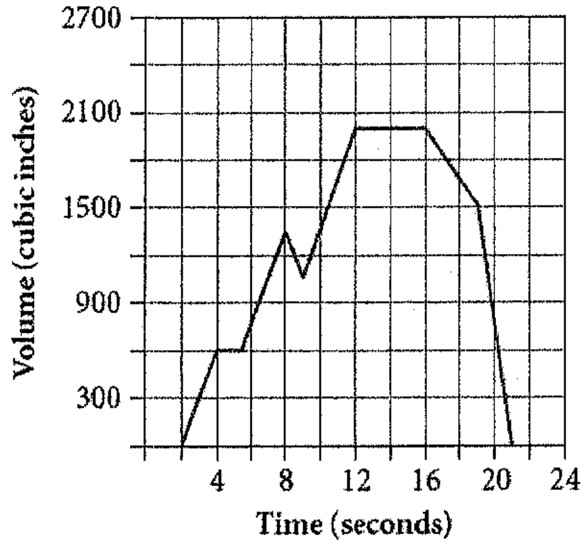
On the graph of this function, you can see the domain values that are possible for the independent variable in this real-world context. This is called the **practical domain**. The practical domain in this example is the set of all instants of time from 0 to 16 hours. We can express this as , where x is the independent variable representing time.

You can also see the values that are possible for the dependent variable. In this example the range is the set of all numbers between 1 ft and about 3.3 ft. We can express this as , where y is the dependent variable representing the depth of the water in feet. Notice that the lowest value for the range (1 ft) does not have to be the starting value when x is zero (2 ft).

The relationship between the independent and dependent variable and the dependent variable is not always a cause and effect relationship. In many situations, time is the independent variable. It is the independent variable in graphs such as population growth or car depreciation and in several relationships of the form (*time, distance*). But time does not *cause* a population to grow or a walker's distance from a given point to change. People do that.

The values of the range depend on the values of the domain. If you know the value of the independent variable, you can determine the corresponding value of the dependent variable. You do this every time you locate a point on the graph of a function.

This graph shows the volume of air in a balloon as it changes over time.



What is the independent variable? How is it measured?

What is the dependent variable? How is it measured?

For what intervals is the volume increasing? What accounts for the increases?

For what intervals is the volume decreasing? What accounts for the decreases?

\

For what intervals is the volume constant? What accounts for this?

Adapted from *Discovering Algebra: An Investigative Approach* by Murdock, Kamischke, and Kamischke

Common Core Math I Unit 3 Functions
Real World Graphs Homework

Name ____________________ Period __________ Date __________

1. For each relationship, identify the independent variable and the dependent variable.

   a. The temperature of a carton of milk and the length of time it has been out of the refrigerator

   b. The weight suspended from a rubber band and the length of the rubber band

   c. The diameter of a pizza and its cost

   d. The number of privately owned cars and the standard of living in a country

   e. The number of cars on the freeway and the level of exhaust fumes in the air

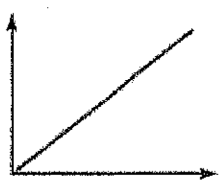
2. Sketch a reasonable graph for each situation and label the axes.

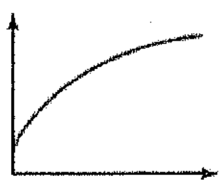
   a. the distance of a person driving in a car from home to work vs. time

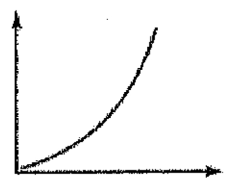
   b. the relationship between a person's age from birth to death and their height

   c. the distance between the ground and a balloon that a small child accidently let go

3. Match each description with its most likely graph, and tell which variable each axis represents.

i. 

ii. 

iii. 

   a. The relationship between your grade on the next math test and the amount of time you spend doing math problems before the test

   b. The relationship between the amount a person earns in an 8-hour day and his or her hourly wage

   c. The change in the area of a square as its side length increases

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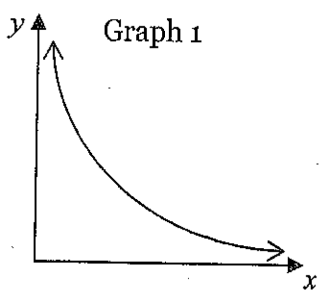
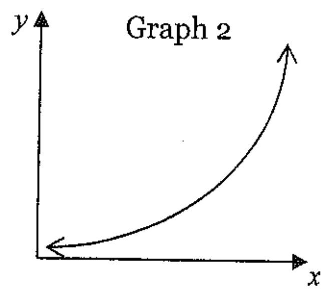
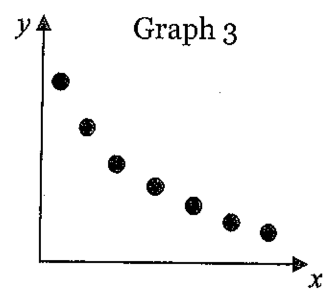
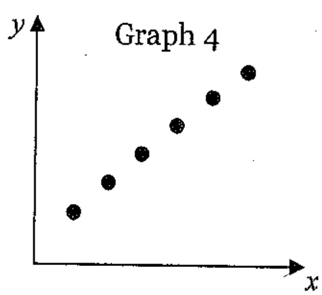
Match each description with its most likely graph. Then label the axes with the appropriate quantities.

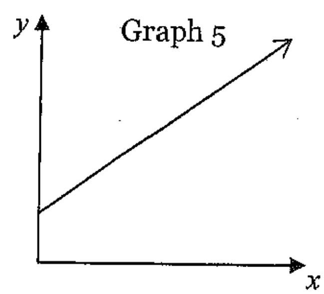
a) the amount of product sold vs. advertising budget
b) the amount of a radioactive substance over time
c) the height of an elevator relative to floor number
d) the population of a city over time
e) the number of students who help decorate for the homecoming dance vs. the time it takes to decorate





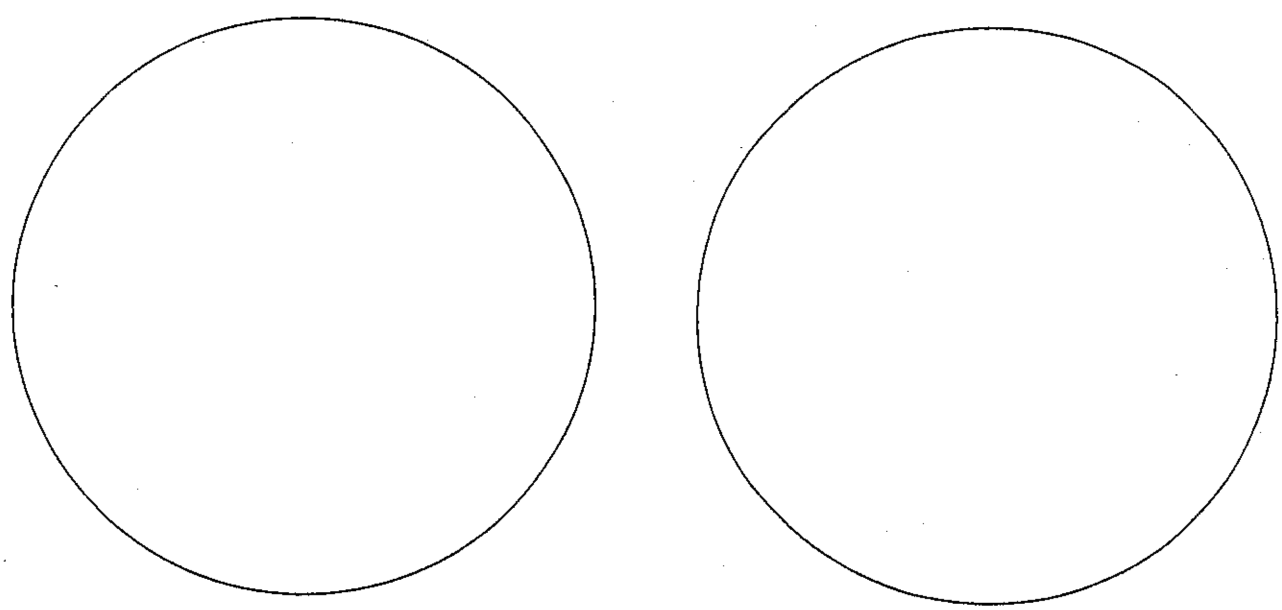






Sort the following key terms into two groups. Then draw lines connecting pairs of terms that go together (one from each group).

dependent, distance, horizontal axis, independent, input, output, time, vertical axis, x, y



Adapted from *Discovering Algebra: An Investigative Approach* by Murdock, Kamischke, and Kamischke